

"This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material. The thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was able to detect it. Although the ring dust is very cold -316F it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now.

"The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009)

Many news reports noted that the ring volume was equal to 1 billion Earths. Is that estimate correct? Let's assume that the ring can be approximated by a washer with an inner radius of r, an outer radius of R and a thickness of h.

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Problem 6 - How does your answer compare to the Press Release information? Why are they different?

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Answer: The area of the large circle is given by π R² minus area of small circle π r² equals $A = \pi$ (R² - r²)

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Answer: Volume = Area x height so $V = \pi (R^2 - r^2) h$

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Answer:
$$V = \pi (R^2 - r^2) h$$

= (3.141) [(1.2 x 10⁷)² - (6.0 x 10⁶)²] 2.4 x 10⁶
= 8.1 x 10²⁰ km³

Note that the smallest number of significant figures in the numbers involved is 2, so the answer will be reported to two significant figures.

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Answer: Volume of a sphere
$$V = 4/3 \pi R^3$$
 so for Earth,
 $V = 1.33 \times (3.14) \times (6.378 \times 10^3)^3$
 $= 1.06 \times 10^{12} \text{ km}^3$

Note that the smallest number of significant figures in the numbers involved is 3, so the answer will be reported to three significant figures.

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Answer: Divide the answer for Problem 3 by Problem 4 to get $8.1 \times 10^{20} \text{ km}^3 / (1.06 \times 10^{12} \text{ km}^3) = 7.6 \times 10^8 \text{ times}$

Problem 6 - How does your answer compare to the Press release information? Why are they different? Answer: The Press Releases say 'about 1 billion times' because it is easier for a non-scientist to appreciate this approximate number. If we rounded up 7.6×10^8 times to one significant figure accuracy, we would also get an answer of '1 billion times'.